Name ________________________ ID3 _ _ _

ECS 315: Probability and Random Processes **2018/1**

HW 7 — Due: Oct 25, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 5 pages.

(b) (1 pt) Work and write your answers **directly on these sheets** (not on other blank sheets of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.

(d) (8 pt) Try to solve all problems.

(e) Carefully write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. For each description of a random variable X below, indicate whether X is a **discrete** random variable.

(a) X is the number of websites visited by a randomly chosen software engineer in a day.

(b) X is the number of classes a randomly chosen student is taking.

(c) X is the average height of the passengers on a randomly chosen bus.

(d) A game involves a circular spinner with eight sections labeled with numbers. X is the amount of time the spinner spins before coming to a rest.

(e) X is the thickness of the longest book in a randomly chosen library.

(f) X is the number of keys on a randomly chosen keyboard.

(g) X is the length of a randomly chosen person's arm.

Problem 2 (Quiz4, 2014)**.** Consider a random experiment in which you roll a 20-sided fair dice. We define the following random variables from the outcomes of this experiment:

$$X(\omega) = \omega, \quad Y(\omega) = (\omega - 5)^2, \quad Z(\omega) = |\omega - 5| - 3$$

Evaluate the following probabilities:

(a) $P[X = 5]$

(b) $P[Y = 16]$

(c) $P[Y > 10]$

(d) $P[Z > 10]$

(e) $P[5 < Z < 10]$

Problem 3. Consider the sample space $\Omega = \{-2, -1, 0, 1, 2, 3, 4\}$. Suppose that $P(A) = |A|/|\Omega|$ for any event $A \subset \Omega$. Define the random variable $X(\omega) = \omega^2$. Find the probability mass function of X.

Problem 4. Suppose X is a random variable whose pmf at $x = 0, 1, 2, 3, 4$ is given by $p_X(x) = \frac{2x+1}{25}$.

Remark: Note that the statement above does not specify the value of the $p_X(x)$ at the value of x that is not 0,1,2,3, or 4.

(a) What is $p_X(5)$?

$=0$

$p_X(0) = 1/25$
$p_X(1) = 3/25$
$p_X(2) = 5/25$
$p_X(3) = 7/25$
$p_X(4) = 9/25$

(b) Determine the following probabilities:

(i) $P[X = 4]$

(ii) $P[X \leq 1]$

(iii) $P[2 \leq X < 4]$

(iv) $P[X > -10]$

Problem 5. The random variable V has pmf

$$p_V(v) = \begin{cases} cv^2, & v = 1,2,3,4, \\ 0, & \text{otherwise.} \end{cases}$$

v	$p_V(v)$
1	c
2	$4c$
3	$9c$
4	$16c$

(a) Find the value of the constant c.

"$\Sigma = 1$" : $c + 4c + 9c + 16c = 1$

$c = \frac{1}{30}$.

$\{1^2, 2^2, 3^2, \dots\} = \{1, 4, 9, 16, \dots\}$

(b) Find $P[V \in \{u^2 : u = 1,2,3,\dots\}]$.

$= P[V \in \{1, 4, 9, 16, \dots\}] = p_V(1) + p_V(4) = c + 16c = 17c = \frac{17}{30}$

(c) Find the probability that V is an even number.

$= p_V(2) + p_V(4) = 4c + 16c = 20c = \frac{20}{30} = \frac{2}{3}$

(d) Find $P[V > 2]$.

$= p_V(3) + p_V(4) = 9c + 16c = 25c = \frac{25}{30} = \frac{5}{6}$

(e) Sketch $p_V(v)$.

(f) Sketch $F_V(v)$. (Note that $F_V(v) = P[V \le v]$.)

Problem 6. The thickness of the wood paneling (in inches) that a customer orders is a random variable with the following cdf:

$$F_X(x) = \begin{cases} 0, & x < \frac{1}{8}, \\ 0.2, & \frac{1}{8} \le x < \frac{1}{4}, \\ 0.9, & \frac{1}{4} \le x < \frac{3}{8}, \\ 1 & x \ge \frac{3}{8}. \end{cases}$$

Determine the following probabilities:

(a) $P[X \le 1/18] = F_X(1/18) = 0$

(b) $P[X \le 1/4] = F_X(1/4) = 0.9$

(c) $P[X \le 5/16] = F_X(5/16) = 0.9$

(d) $P[X > 1/4] = \quad = 1 - P[X \le 1/4] = 1 - F_X(1/4) = 1 - 0.9 = 0.1$

(e) $P[X \le 1/2] = F_X(1/2) = 1$

[Montgomery and Runger, 2010, Q3-42]

Remark: Try to calculate these values directly from the cdf. (Avoid converting the cdf to pmf first.)